Appendix E: Sample Laboratory Report

There is no set length for a problem report but experience shows good reports are typically four pages long. Graphs and photocopies of your lab journal make up additional pages. Complete reports will include the terminology and the mathematics relevant to the problem at hand. Your report should be a clear, concise, logical, and honest interpretation of your experience. You will be graded based on how well you demonstrate your understanding of the physics. Because technical communication is so important, neatness, and correct grammar and spelling are required and will be reflected on your grade.

Note: As with Problem 1 of Lab 1, the double vertical bars indicate an explanation of that part of the report. These comments are not part of the actual report.

Usually a lab report consists of the following parts:

- Title,

- Statement of the Problem,

- Prediction,

- Experiment and Results,

- Conclusions,

- References.

In this appendix there are three sample lab reports, which are based on the actual lab reports written by students who took this course before. Our main goal of putting them here is to give you some information about writing lab reports. Basically, any lab report is a simplified version of a research paper or technical report. Therefore, it is written in a scientific style and it has the same main parts as a research paper/technical report. As you go from #1 to #3, you'll notice the improvement in the quality of the samples. This is typical level of improvement of the students taking this course. Your goal is to be able to write such lab reports as sample #3 after completing this course.

It's useful to read the sample lab reports after completing the following laboratories in class:

- Sample # 1 after Projectile Motion;

- Sample #2 after Conservation of Energy;

- Sample #3 after Rotational Motion.

Sample Lab Report #1

Title

Write a descriptive title with your name, names of partners, date performed and TA name.

Projectile Motion

September 27, 2003

Performed by: **Roman Ivanov**, Tom Johnson and Mun Tiang

TA name: Ken Spillner

Statement of the problem

In a sentence or two, state the problem you are trying to solve. List the equipment you will use and the reasons for selecting such equipment.

The problem was to determine the dependence of the time of flight of a projectile on its initial horizontal velocity. We rolled an aluminum ball down a ramp and off the edge of a table starting from rest at two different positions along the ramp. Starting from the greater height up the ramp meant the ball had a larger horizontal velocity when it rolled along the table. Since the table was horizontal, that was the horizontal velocity when it entered the air. See Figure 1 from my lab journal for a picture of the set-up.

We made two movies with the video equipment provided, one for a fast rolling ball and one for a slower one. These movies were analyzed with LabVIEW™ to study the projectile's motion in the horizontal and vertical directions.

Prediction

This is the part of the lab where you try to predict the outcome of the experiment based on your general knowledge of Physics. If your predictions were wrong and you understood it during the lab, write correct ones in your lab report. Also, attach initial predictions to your lab report and explain what was wrong. If you still have problems with the prediction, ask your TA or go to TA office hours for some help. Generally, predictions are based on the fundamental laws or principles. Therefore, refer to these laws as a starting point of your predictions.

Our group predicted that the time the ball took to hit the ground once it left the table would be greater if the horizontal velocity were greater. We have observed that the faster a projectile goes initially, the longer its trajectory. Since the gravitational acceleration is constant, we reasoned that the ball would take more time to travel a larger distance. However, after completing the lab we understood that horizontal and vertical motion is independent. Therefore, for two given cases where the object had different initial horizontal velocity, the time of flight was the same. So, our initial predictions were wrong.

Presentation of your lab report is an important part of the evaluation process. It has to be written in a clear and understandable way for your peers, TAs and Professors. **We recommend using Microsoft Word for writing lab reports, especially for its handy tool - Equation Editor. Equation Editor allows you to type complicated mathematical expressions in a compact and elegant way. In order to use this tool go to Insert Menu → Object → Microsoft Equation 3.0.**

Mathematically, we start from the definition of acceleration:

$$a = \frac{d}{dt}\left(\frac{dy}{dt}\right)$$

and integrate twice with respect to time to see how a change in time might be related to initial velocity. We found that:

$$y - y_o = v_o\Delta t + 0.5a\Delta t^2 \quad (1)$$

In this example, the student is explaining incorrect predictions. Although motion is two-dimensional, he/she is using $\boldsymbol{u}_0$ while correct notation is $\boldsymbol{u}_{0y}$ as it is a projection of velocity onto y-axis.

With the y-axis vertical and the positive direction up, we know the acceleration is -g. We also know that vo is the initial velocity, and yo - y is h, the height of the table. Solving for Δt one finds:

$$\Delta t = \frac{v_o \pm \sqrt{(v_o^2 + 2hg)}}{g} \quad (2)$$

Faced with a choice in sign, our group chose the solution with the positive sign, deciding that a possible negative value for elapsed time does not correspond with our physical situation.   From equation (2), we deduced that if vo increased, then the time of fall also increases. This coincided with our prediction that a projectile with fastest horizontal velocity would take the most time to fall to the ground. For a graph of our predicted time of flight versus initial horizontal velocity, see Graph A from the lab journal.

LabVIEW™ generated graphs of x and y positions as functions of time. Our prediction for the vertical direction was equation (1). Since the ball only has one acceleration, we predicted that equation (1) would also be true for the horizontal motion:

$$x - x_o = v_o\Delta t + 0.5a\Delta t^2$$

The dotted lines on the printed graphs represent these predictions.

The Example of Two Bullets

Our TA asked us to compare a bullet fired horizontally from a gun to a bullet dropped vertically. Our group decided the bullet that is fired horizontally will take longer to hit the ground than the one that is simply dropped from the same height.

Experiment and results

This section describes your experimental method, the data that you collected, any problems in gathering the data, and any crucial decisions you made. Your actual results should show you if your prediction was correct or not.

To ensure the ball's velocity was completely horizontal, we attached a flat plank at the end of the ramp. The ball rolls down the ramp and then goes onto the horizontal plank. After going a distance (75 cm) along the plank, the ball leaves the edge of the table and enters projectile motion.

We measured the time of flight by simply counting the number of video frames that the ball was in the air. The time between frames is 1/30 of a second since this is the rate a video camera takes data. This also corresponds to the time scale on the LabVIEW™ graphs. We decided to compare the times of flight between a ball with a fast initial velocity and one with a slow initial velocity. To get a fast velocity we started the ball at the top of the ramp. A slower velocity was achieved by starting the ball almost at the bottom of the ramp.

During the time the ball was in the air, the horizontal velocity was a constant, as shown by the velocity in the x-direction graphs for slow and fast rolling balls. From these graphs, the slowest velocity we used was 1.30 m/s, and the fastest was 2.51 m/s.

After making four measurements of the time of flight for these two situations, we could not see any correspondence between time of flight and initial horizontal velocity (see table 1 from lab journal). As a final check, we measured the time of flight for a ball that was started approximately halfway up the ramp and found it was similar to the times of flight for both the fast and the slow horizontal velocities (see table 2 from lab journal).

A discussion of uncertainty should follow all measurements. No measurement is exact. Uncertainty must be included to indicate the reliability of your data.

Most of the uncertainty in recording time of flight came from deciding the time for the first data point when the ball is in the air and the last data point before it hit the ground. We estimated that we could be off by one frame, which is 1/30 of a second. To get a better estimate of this uncertainty, we repeated each measurement four times. The average deviation served as our experimental uncertainty (see Table 1 from lab journal). This uncertainty matched our estimate of how well we could determine the first and the last frame of the projectile trajectory.

Conclusions

This section summarizes your results. In the most concise manner possible, it answers the original question of the lab.

Our graph indicates that the time of flight is independent of the ball's initial horizontal velocity (see lab journal, Graph A). We conclude that there is no relationship between these two quantities.

A good conclusion will always compare actual results with the predictions.   If your prediction was incorrect, then you must discuss where your reasoning went wrong. If your prediction was correct, then you should review your reasoning and discuss how this lab served to confirm your knowledge of the basic physical concepts.

Our prediction is contradicted by the apparent independence of the time of flight and initial horizontal velocity. We thought that the ball would take longer to fall to the floor if it had a greater initial horizontal velocity. After some discussion, we determined the uncertainty in our prediction. We did not understand that the vertical motion is completely independent of the horizontal motion. Thus, in the vertical direction the equation

$$y - yo = vo\Delta t + 0.5a\Delta t^2$$

means that the vo is the only the y-component of initial velocity. Since the ball rolls horizontally at the start of its flight, vo in this equation always equals zero.

The correct equation for the time of flight, with no initial vertical component of velocity, is actually:

$$y - yo = 0.5a\Delta t^2$$

In this equation, there is no relationship between time of flight and initial horizontal velocity.

Furthermore, the graphs we generated with LabVIEW™ showed us that velocity in the y-direction did not change when the initial horizontal velocity changed.   Velocity in the y-direction is always approximately zero at the beginning of the trajectory. It is not exactly zero because of the difficulty our camera had determining the position when the projectile motion begins. We observed that the y-velocity changed at the same rate (slope of v_y plots, graphs 1 and 2) regardless of the horizontal velocity. In other words, the acceleration in the y-direction is constant, a fact that confirms the independence of vertical and horizontal motion.

After you have compared your predictions to your measured results, it is helpful to use an alternative measurement to check your theory with the actual data. This should be a short exercise demonstrating to yourself and to your TA that you understand the basic physics behind the problem. Most of the problems in lab are written to include alternative measurements. In this case, using the time of fall and the gravitational constant, you can calculate the height of the table.

The correct equation for the horizontal motion is

$$x - x_o = v_o\Delta t$$

The horizontal acceleration is always zero, but the horizontal distance that the ball covers before striking the ground does depend on initial velocity.

Alternative Analysis

Since yo - y = h and a = -g we can check to see if our measured time of flight gives us the height of the table. From our graph, we see that the data overlaps in a region of about 0.41 sec. With this as our time of flight, the height of the table is calculated to be 82.3 cm. Using a meter stick, we found the height of the table to be 80.25 cm. This helped convince us that our final reasoning was correct.

The example of the two bullets discussed in the Prediction section was interpreted incorrectly by our group. Actually, both bullets hit the ground at the same time. One bullet travels at a greater speed, but both have the same time of flight. Although this seems to violate "common sense" it is an example of the independence of the horizontal and vertical components of motion.

References

List books, journals or any other resources that you used to write your lab report..

1. Tipler, Paul A. Physics for Scientists and Engineers. 4th, W. H. Freeman: 1999.

The following are pages photocopied from my lab journal:

Sample Lab Journal   July 13ᵗʰ, 1998

h = height of table

H = ball's starting height

Equipment: ramp, aluminum ball, video camera, computer, meterstick

Purpose: Determine if Time of flight depends on initial horizontal velocity

Method: One person releases ball, another operates video camera. Count frames with ball in the air to find time of flight.

Exploration: Larger H results in larger horizontal $\vec{v}$.

Table 1
Fast Ball, $H \simeq 35$cm, $v_0 = 2.51$ m/s

Trial	Frames with ball in air	Deviation from ave.
1	13	0.25
2	13	0.25
3	12	0.75
4	13	0.25
Average	12.75	0.4

Time of flight:
12.75 frames $\times \frac{1}{30}$ sec/frame =

$\underline{0.42 \text{ sec}}$

Uncertainty:
± 0.4 frames $\times \frac{1}{30}$ sec/frame =

$\underline{\pm 0.014 \text{ sec}}$

Slow Ball, $H \simeq 10$cm, $v_0 = 1.3$ m/s

1	12	0.5
2	13	0.5
3	13	0.5
4	12	0.5
Average	12.5	0.5

Time of flight:
12.5 frames $\times \frac{1}{30}$ sec/frame =

$\underline{0.41 \text{ sec}}$

Uncertainty
$\pm 0.5 \times \frac{1}{30}$ sec/frame =

$\underline{\pm 0.018 \text{ sec}}$

Table 2
Medium Ball, $H \simeq 20$, $v_0 = 1.8$ m/s

Trial	# frames	Deviation
1	12	0.5
2	12	0.5
3	13	0.5
4	13	0.5
Average	12.5	0.5

Time of flight:
12.5 frames $\times \frac{1}{30}$ sec/frames =

$\underline{0.41 \text{ sec}}$

Uncertainty: $\pm 0.5 \times \frac{1}{30}$ sec/frame =

$\underline{\pm 0.018 \text{ sec}}$

Graph A: Time of flight vs. Initial Average Horizontal $\vec{v}$

Time of flight (sec)

Initial Average Horizontal $\vec{v}$ (m/s)

Predicted curve — — — — —

results: ⊢•⊣
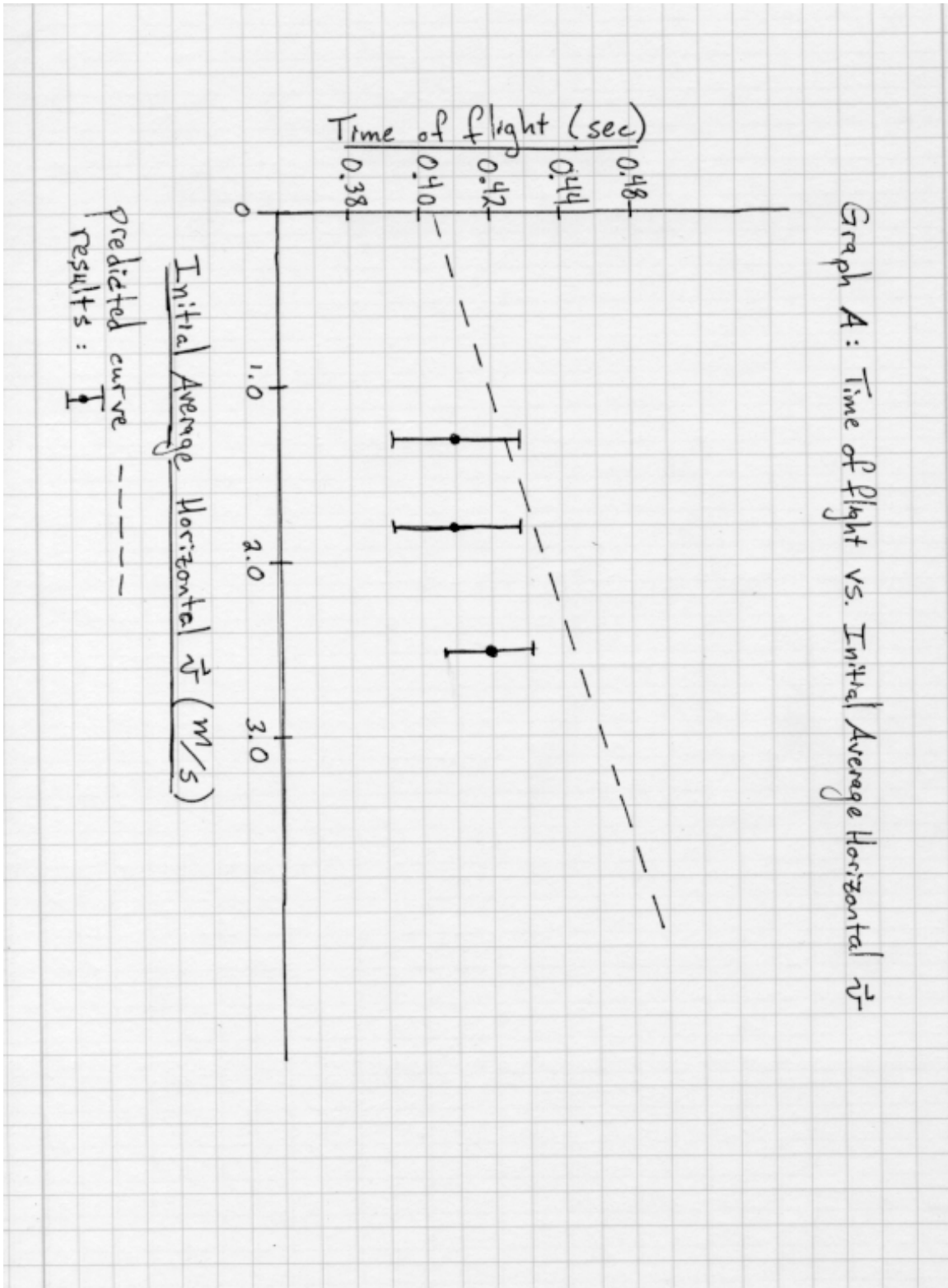
Movie Name: "Graphs of position and time for ball with slow initial x-velocity."

Data Taken: Tuesday, July 13, 1999, 10:17 AM
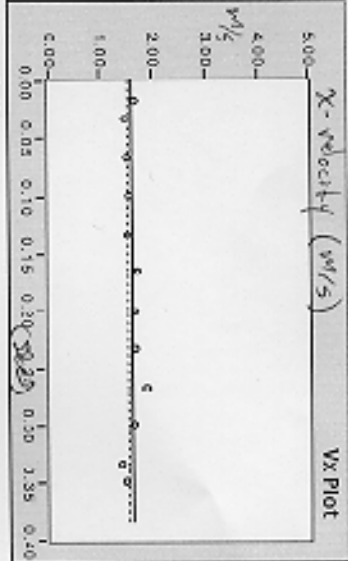
Your Graph Title Goes Here!
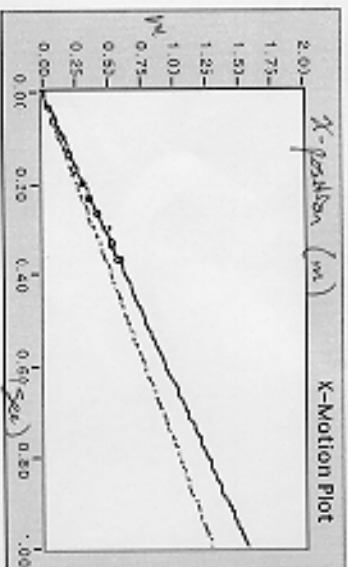
X-Motion Plot

X-position (m)

X Fit

$x(t) = 0.00 + 1.57t$

X Prediction

$x(t) = 0.00 + 1.30t$

Y-Motion Plot

y-position (m)

Y Fit

$y(t) = 0.25 + 1.19t + -9.45t^2$

Y Prediction

$y(t) = 0.00 + 0.00t$

Vx Plot

X-velocity (m/s)

Vx Fit

$Vx(t) = 1.63 + 0.00t$

Vx Prediction

$Vx(t) = 1.53 + 0.00t$

Vy Plot

y-velocity (m/s)
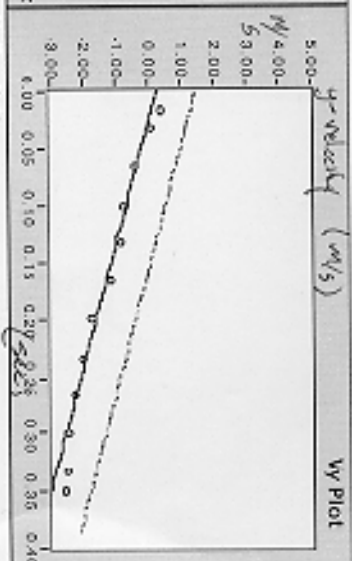
Vy Fit

$Vy(t) = 0.30 + -9.50t$

Vy Prediction

$Vy(t) = 1.50 + -9.40t$

X-velocity = 1.50 m/s
Uncertainty range
± 0.08 m/s

y-acceleration:
= -9.46 m/s²
Uncertainty Range:
± 1.0 m/s²

Movie Name: " Graphs of position and velocity for ball with test initial x-velocity."
Data Taken: Tuesday, July 13, 1999, 10:54 AM

Your Graph Title Goes Here!

X-Motion Plot
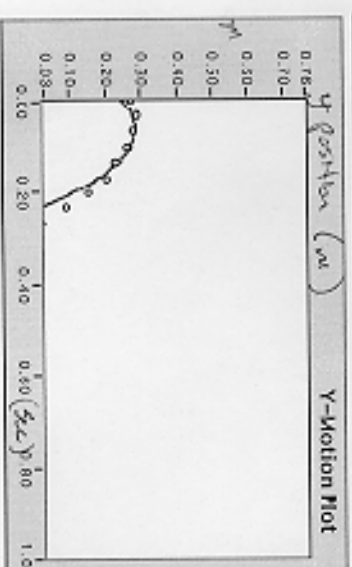
X Prediction
x(t) = 0.00 + 2.50t

X Fit
x(t) = 0.00 + 2.30t

Vx Prediction
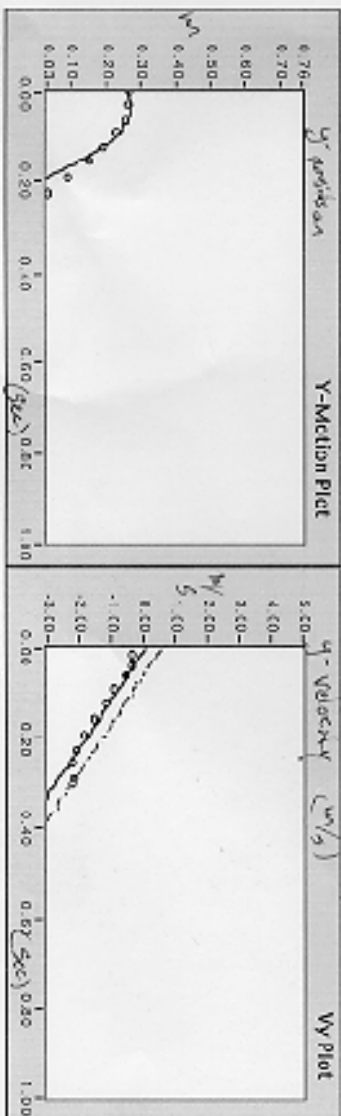Vx(t) = 2.40 + 0.00t

Vx Fit
Vx(t) = 2.40 + 0.00t

Y Prediction
y(t) = 0.00 + -9.80t^2

Y Fit
y(t) = 0.26 + 0.60t + -9.50t^2

Y-Motion Plot

Vy Prediction
Vy(t) = 0.60 + -9.30t

Vy Fit
Vy(t) = 0.10 + -9.30t

Vy Plot
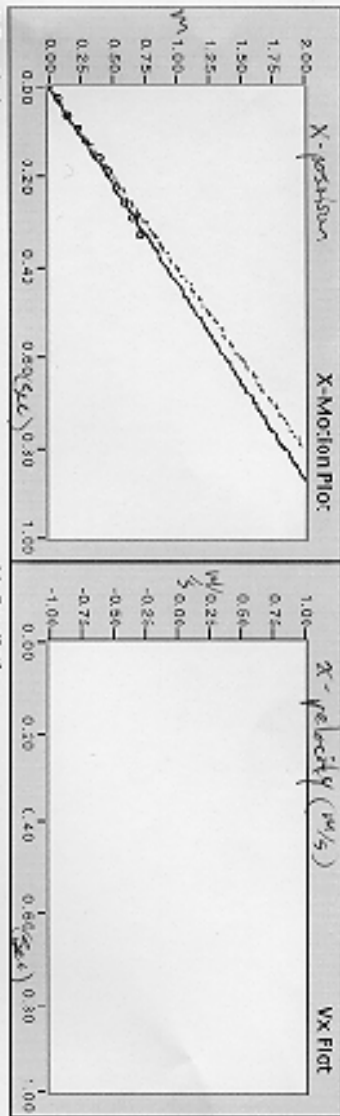
X-velocity = 2.40 $\frac{m}{s}$
Uncertainty Range:
± 0.08 $\frac{m}{s}$

Y-acceleration = -9.30 $\frac{m}{s^2}$
Uncertainty Range:
± 1.0 $\frac{m}{s^2}$

Title

Write a descriptive title with your name, names of partners, date performed and TA name.

Efficiency of the bumper derived using inclined track
October 16, 2003

Performed by: **Roman Ivanov**, Tom Johnson and Mun Tiang

TA name: Ken Spillner

Statement of the problem

This is an introduction to the problem. In a paragraph or two, state the problem you are trying to solve and what you expect to learn from it. List the equipment you will use and reasons for selecting such equipment.

In this lab our goal was to estimate the efficiency of the magnetic bumper. To do so we used an inclined track experiment. Our group released the cart from certain height and took a video of the carts motion before and after collision. The acceleration of the cart up and down the ramp was determined using video analysis. Using this data we were able to calculate efficiency of the magnetic bumper.

Prediction

This is the part of the lab where you try to predict the outcome of the experiment based on your general knowledge of Physics. If your predictions were wrong and you understood it during the lab, write correct ones in your lab report. Also, attach initial predictions to your lab report and explain what was wrong. If you still have problems with the prediction, ask your TA or go to TA office hours for some help.

Generally, predictions are based on the fundamental laws or principles. Therefore, refer to these laws as a starting point of your predictions (For example, if you are dealing with dynamics and forces, you can mention Newton's Second Law as a starting point).

Presentation of your lab report is an important part of the evaluation process. It has to be written in a clear and understandable way for your peers, TAs and Professors. **We recommend using Microsoft Word for writing lab reports, especially for its handy tool - Equation Editor. Equation Editor allows you to type complicated mathematical expressions in a compact and elegant way. In order to use this tool go to Insert Menu → Object → Microsoft Equation 3.0.**

To find the efficiency of magnetic bumper we used conservation of energy:

$$E_{before} = E_{after} + E_{bumper}$$

Kinetic energy before (E_{before}) and after (E_{after}) collision is determined by potential energy and work done by the frictional force: $E_{before} = mgh_i - W_{before}$, $E_{after} = mgh_f + W_{after}$, where W_{before} and W_{after} is work done by the frictional force before and after collision, m is mass of the cart, h_i, h_f are initial and final heights (see Fig. 1 below). The efficiency of the bumper is calculated using the following formula

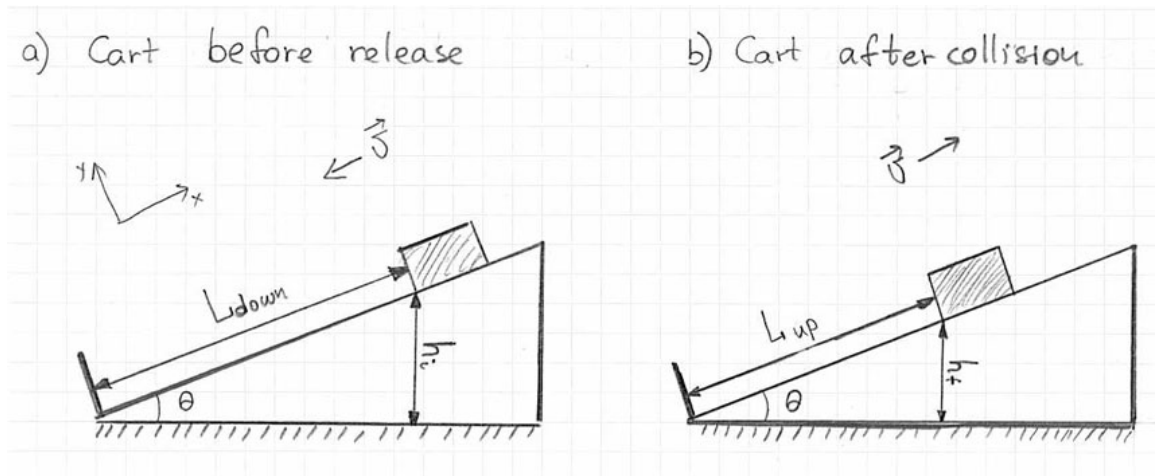$$eff = \frac{E_{after}}{E_{before}}.$$

Fig. 1. Cart going down the track, bounces with the bumper and going back.

Initial and final potential energies can be determined experimentally by measuring the initial and final heights. Work done by the friction is found assuming that the frictional force is constant up and down the ramp: $W_{before} = F_{friction}L_{down}$, $W_{after} = F_{friction}L_{after}$, where L_{down}, L_{up} are distances down (before collision) and up (after collision) the ramp. The distances are known, so we have to derive the expression for the force of friction in terms of the measured quantities.

Let's apply Newton's Second Law to two cases: when the cart is moving down the track and up. See Fig. 2 for the Force diagrams.
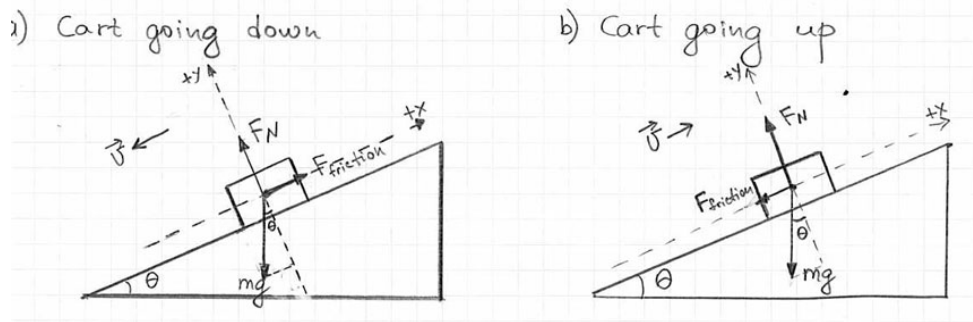
$$\sum_i \vec{F}_i = m\vec{a}$$

Fig. 2. Force diagram for two cases: cart going down and up. Notice the difference in the force of friction.

In rotated system of coordinates $\sum_i F_{iy} = 0$ and $\sum_i F_{ix} = ma_x$. From here we concluded that we can find force of friction in terms of the measured accelerations. Taking into account that the force of friction always opposes the motion, we find the following relations for the motion up and down the ramp:

$$\begin{cases} ma_{up} = mg \sin q + F_{friction} \\ ma_{down} = mg \sin q - F_{friction} \end{cases} \Rightarrow F_{friction} = m(a_{up} - a_{down})/2$$

Acceleration up and down the ramp can be determined from video analysis. Substituting above expressions for E_{after} and E_{before} , efficiency of the magnetic bumper is

$$eff = \frac{mgh_f + \dfrac{m}{2}(a_{up} - a_{down}) * L_{up}}{mgh_i - \dfrac{m}{2}(a_{up} - a_{down}) * L_{down}}$$

After simplifying our group got the following expression:

$$eff = \frac{L_{up}}{L_{down}} * \frac{g \sin q + (a_{up} - a_{down})/2}{g \sin q - (a_{up} - a_{down})/2}$$

Experiment and results

This section describes your experimental method, the data that you collected, any problems in gathering the data, and any crucial decisions you made. Briefly discuss your setup and explain how you collected your data. Present information in a clear way either in tables or graphs. Make sure you have correct units shown on the graphs. Include all information that you used in the lab to calculate some final results so that we can repeat the calculations and see whether this step is correct or not. In order to arrive at a valid conclusion in testing a theory or hypothesis it is necessary to understand the underlying concepts of measurement uncertainties. Calculate your uncertainties and indicate the

largest contribution to uncertainty. Finally, compare the experimental results and theoretical prediction. Discuss any agreement or disagreement between them.

In order to find the efficiency of the bumper we released the cart from initial height h_i and took the video of the motion. Also, we determined the final height the cart reached. For practical reasons it is easier to measure the distances from the bumper which are related to the height $h = L\sin q$. Then we analyzed the video and figured out the acceleration before and after collision by fitting the position versus time curve. Experimental data is given in Table 1 and Table 2:

Before collision	After collision
L_i =80 cm	L_f =64 cm
a_{down} =39 cm/s²	a_{up} =44 cm/s²
q =3°	q =3°

Table. 1. Summary of experimental results.

Having necessary information from our experiment we are ready to calculate the efficiency:

$$eff = \frac{0.64m}{0.8m} * \frac{9.88 \frac{m}{s^2} * \sin 3° + \frac{1}{2}(0.44\frac{m}{s^2} - 0.39\frac{m}{s^2})}{9.88 \frac{m}{s^2} * \sin 3° - \frac{1}{2}(0.44\frac{m}{s^2} - 0.39\frac{m}{s^2})} = 0.85$$

So the efficiency of the bumper is 85%. The uncertainty in this experiment basically comes from measuring the distance (uncertainty of the meter ± 0.5 cm), from analyzing the motion (uncertainty related to the video analysis can be estimated by clicking on the same point on the car's image and recording its position) and fitting the experimental curve (uncertainty caused by the fitting of experimental points can be estimated by finding the largest deviation of the experimental point from the fitted curve or by varying the constants, for example acceleration, so that your equation still fits the data). Our group estimated uncertainty to be around 10%.

For more information on uncertainty see Appendix B.

Conclusions

This section summarizes your results. In the most concise manner possible, it answers the original question of the lab. A good conclusion will always compare actual results with the predictions. If your prediction was incorrect, then you must discuss where your reasoning went wrong. If your prediction was correct, then you should review your reasoning and discuss how this lab served to confirm your knowledge of the basic physical concepts.

In this lab we determined the efficiency of a magnetic bumper. After completing the experiment we concluded that magnetic bumper is very efficient (85%). Our conclusion is consistent with the previous lab results. Results check – efficiency of the bumper is indeed less than 100%, so our results make sense in terms of energy conservation.

References

List books, journals or any other resources that you used to write your lab report.

1. Tipler, Paul A. Physics for Scientists and Engineers. 4th, W. H. Freeman: 1999.
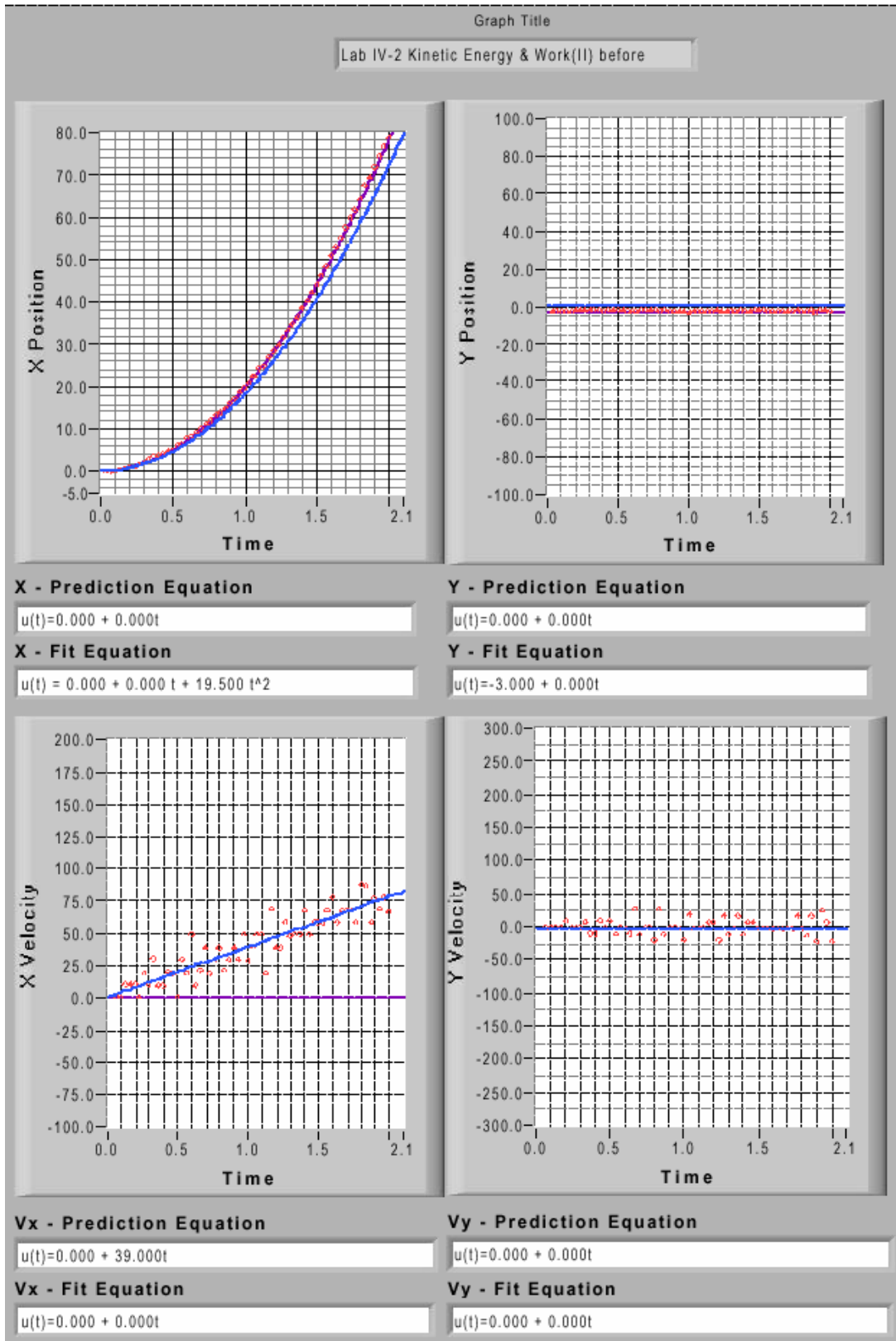
Graph Title

Lab IV-2 Kinetic Energy & Work(II) before

X - Prediction Equation

u(t)=0.000 + 0.000t

X - Fit Equation

u(t) = 0.000 + 0.000 t + 19.500 t^2

Y - Prediction Equation

u(t)=0.000 + 0.000t

Y - Fit Equation

u(t)=-3.000 + 0.000t

Vx - Prediction Equation

u(t)=0.000 + 39.000t

Vx - Fit Equation

u(t)=0.000 + 0.000t

Vy - Prediction Equation

u(t)=0.000 + 0.000t

Vy - Fit Equation

u(t)=0.000 + 0.000t

Table 2. Experimental results from video analysis.

Title

Write a descriptive title with your name, names of partners, date performed and TA name.

Oscillation Frequency with Two Springs
June 7, 2004

Performed by: **Roman Ivanov**, Tom Johnson and Mun Tiang

TA name: Ken Spillner

Statement of the problem

This is an introduction to the problem. In a paragraph or two, state the problem you are trying to solve and what you expect to learn from it. List the equipment you will use and reasons for selecting such equipment.

The problem was to determine the frequency of oscillation of the cart attached to two springs with different spring constants. Our group built a laboratory model as shown below:
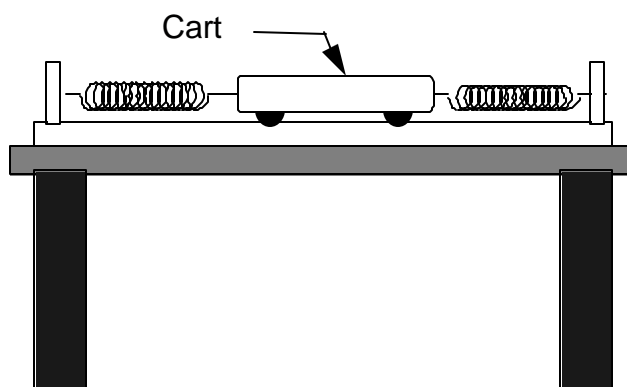

Fig. 1. Experimental setup: cart attached to two springs with different spring constants.

We displaced the cart from equilibrium and then gently released it. The period of the oscillation was determined using a stop watch and video analysis. The cart was selected because it rolls with low friction; neglecting friction simplified our analysis. Video analysis provides precise measurements of the period, and the stopwatch provides an alternative method of checking the period.

Prediction

This is the part of the lab where you try to predict the outcome of the experiment based on your general knowledge of Physics. If your predictions were wrong and you understood it during the lab, write correct ones in your lab report. Also, attach initial predictions to your lab report and explain what was wrong. If you still have problems with the prediction, ask your TA or go to TA office hours for some help.

Generally, predictions are based on the fundamental laws or principles. Therefore, refer to these laws as a starting point of your predictions (For example, if you are dealing with dynamics and forces, you can mention Newton's Second Law as a starting point).

Presentation of your lab report is an important part of the evaluation process. It has to be written in a clear and understandable way for your peers, TAs and Professors. **We recommend using Microsoft Word for writing lab reports, especially for its handy tool - Equation Editor. Equation Editor allows you to type complicated mathematical expressions in a compact and elegant way. In order to use this tool go to Insert Menu → Object → Microsoft Equation 3.0.**

To find the frequency of oscillation we have to determine equation that governs the dynamics of the system. Let's start with Newton's Second Law and find forces acting on the cart.

$$\sum_i \vec{F}_i = m\vec{a} \tag{1}$$

The cart only moves horizontally, so we can ignore vertical forces. There are two forces acting on the cart in the horizontal direction: $\vec{F}_1$ and $\vec{F}_2$ (see Fig. 2), where $\vec{F}_1$ and $\vec{F}_2$ are restoring forces obeying Hooke's Law: $F = -k\Delta x$, k - spring constant, Δx - stretch of spring.
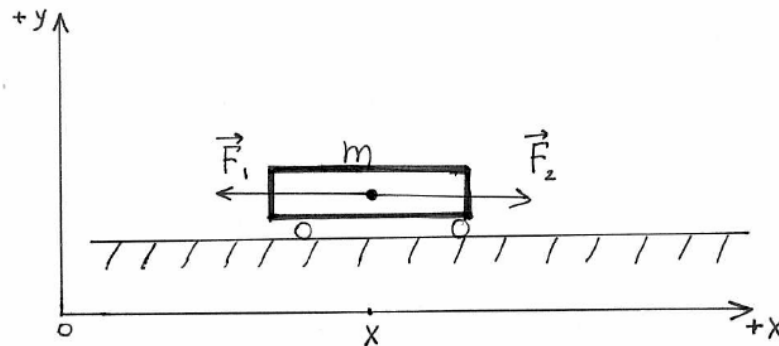

Fig. 2. Forces acting on the cart attached to two springs, x being position of the cart. Friction is ignored since the cart rolls nearly without friction.

At the equilibrium position x_0 forces are equal: $F_2{}^{eq} = F_1{}^{eq}$. As the cart is displaced from the equilibrium position forces change, which determines the motion of the cart. Therefore, from now on we are concerned with the change in forces with respect to equilibrium ones (here it's useful to make analogy with potential energy – only change in potential energy matters).

Let's consider the situation when the cart is displaced to the **right** from equilibrium (see Fig. 3) and let's write Newton's Second Law:

$$(\vec{F}_1^{\;eq} + \Delta\vec{F}_1) + (\vec{F}_2^{\;eq} + \Delta\vec{F}_2) = m\vec{a}$$

$$\vec{F}_1^{\;eq} + \vec{F}_2^{\;eq} = 0 \Rightarrow \Delta\vec{F}_1 + \Delta\vec{F}_2 = m\vec{a} \tag{2}$$

where $\vec{F}_1^{\;eq}$ and $\vec{F}_2^{\;eq}$ are forces exerted by the springs at the equilibrium, $\Delta\vec{F}_1$ and $\Delta\vec{F}_2$ are changes in forces with respect to equilibrium.
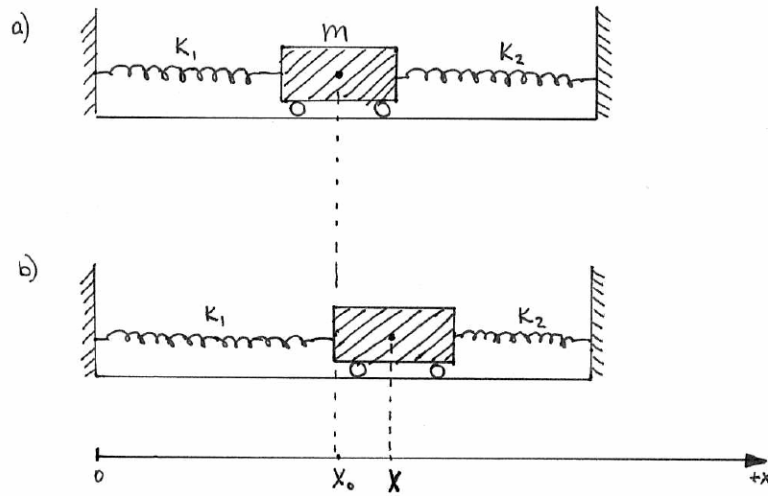

Fig. 3. Cart at equilibrium position a) and cart displaced from equilibrium position b).

By projecting equation (2) onto x-axes at the moment when cart is displaced to the right, we get

$$\Delta F_{1x} + \Delta F_{2x} = m\frac{d^2 x}{dt^2} \tag{3}$$

The changes in forces ΔF_{1x} and ΔF_{2x} can be found using Hook's Law

$$\Delta F_{1x} = F_{1x} - F_{1x}^{eq} = -k_1(x - x_0), \; \Delta F_2 = F_{2x} - F_{2x}^{eq} = -k_2(x - L_2) + k_2(x_0 - L_2) = -k_2(x - x_0),$$

where L_2 is the relaxed length of the spring.

Finally we got equation that describes the motion of the cart:

$$\frac{d^2 x}{dt^2} + \frac{(k_2 + k_1)}{m}(x - x_0) = 0 \tag{4}$$

Since we know that motion of the cart is periodic around its equilibrium position, equation (4) should describe periodic motion. We can guess a solution of Eq. (4) as a periodic function: $x(t) = A\sin(wt + d) + x_0$. If this solution works, we know that we can use it to find the oscillation frequency f, because $w = 2pf$. For this to be a solution, it must describe motion of the cart and also satisfy Eq. 4 for any moment of time. To compare with equation (4), take derivatives

$$x(t) = x_0 + A\sin(wt + d) \Rightarrow \frac{dx}{dt} = Aw\cos(wt + d) \Rightarrow \frac{d^2x}{dt^2} = -Aw^2\sin(wt + d)$$

and notice that:

$$\frac{d^2x}{dt^2} + w^2(x - x_0) = 0 \tag{5}$$

Comparing Eq. 5 with Eq. 4 , it will work if $w^2 = \dfrac{k_1 + k_1}{m}$. Therefore we have a solution of Eq. 4:

$x = A\sin(\sqrt{\dfrac{k_2 + k_1}{m}}t + d)$ with oscillation frequency:

$$w = 2pf = \sqrt{\frac{k_2 + k_1}{m}} \ .$$

To compare with experiment, it is useful to note the period: $T = \dfrac{2p}{w} = 2p\sqrt{\dfrac{m}{k_2 + k_1}}$.

So, based on our predictions, period is inversely proportional to the sum of two spring constants. If we attach two identical springs to the cart, period of oscillations should decrease (be multiplied by a factor of $\dfrac{1}{\sqrt{2}}$) with respect to the period of one spring attached.

Experiment and results

This section describes your experimental method, the data that you collected, any problems in gathering the data, and any crucial decisions you made. Briefly discuss your setup and explain how you collected your data. Present information in a clear way either in tables or graphs. Make sure you have correct units shown on the graphs. Include all information that you used in the lab to calculate some final results so that we can repeat the calculations and see whether this step is correct or not. In order to arrive at a valid conclusion in testing a theory or hypothesis it is necessary to understand the underlying concepts of measurement uncertainties. Calculate your uncertainties and indicate the largest contribution to uncertainty. Finally, compare the experimental results and theoretical prediction and discuss the agreement between them.

Our group started from a simple experiment: we took measurements of the period of the oscillating cart with a stop watch and got $T = 1.5$ s. However, our measurements with the stop watch have large uncertainty and therefore can be used as an estimate only. Since there is a delay in time due to human reaction, we estimated uncertainty to be $\pm$ 0.5 s. As we can see, there is large uncertainty involved in this simple experiment.

A discussion of uncertainty should follow all measurements. No measurement is exact. Uncertainty must be included to indicate the reliability of your data.

We decided to take another approach then. Our group made a video of the motion and analyzed it with the help of LabView software. Using video analysis we can substantially reduce uncertainty and get better results. See attached Fig. 5 with results of video analysis. Uncertainty in LabView program is hard to estimate. Mainly, it comes from fitting the experimental curve and some displacement of

pointing dot during video analysis. One can estimate the uncertainty for this experiment by looking at the variation of the period on the experimental graph.

Uncertainty of the experimental data is specific to the experiment performed. In general, uncertainty related to the video analysis can be estimated by clicking on the same point on the car's image and recording its position. Uncertainty caused by the fitting of experimental points can be estimated by finding the largest deviation of the experimental point from the fitted curve or by varying the constants, for example acceleration, so that your equation still fits the data.

Our group estimated uncertainty to be $\pm$ 10%. After analyzing the video, we found the experimental frequency to be $f = 0.745 \pm 0.08$ Hz.

To be able to compare theoretical results with experimental, we had to get the spring constants. In order to do so we attached a 50-gram weight to each of the springs and measured the displacement from relaxed lengths (L_1 and L_2 were equal to 6 cm). See Fig 4. The uncertainty in the measurements of the spring lengths is ± 0.5 cm.

Spring 1:   Spring 2:

$\Delta x_1 = 17 \pm 0.5$ cm $\Delta x_2 = 18 \pm 0.5$ cm

$k_1 = \dfrac{mg}{\Delta x_1} \Rightarrow k_1 = 2.88$ N/m $k_2 = \dfrac{mg}{\Delta x_2} \Rightarrow k_2 = 2.72$ N/m
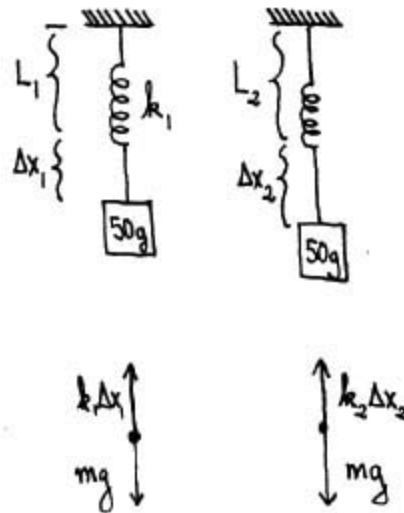

Fig. 4. Diagram for calculation of the spring constants.

Based on the information gathered, we calculated the theoretical frequency using formula

$$f = \frac{1}{2p}\sqrt{\frac{k_2 + k_1}{m}},$$

where k_1 and k_2 are spring constants, $m = 251.7 \pm 10$ g is mass of the cart. Predicted frequency is $f = 0.751$ Hz ± 0.03 Hz. Calculated frequency is within uncertainty of our experiment, so we conclude that our prediction adequately describe experimental results.

During experiment we noticed that frequency of oscillations is slightly changing with time and it can be explained by effect of friction on the motion of the cart.

Conclusions

This section summarizes your results. In the concise manner, it answers the original question of the lab. A good conclusion will always compare actual results with the predictions. If your prediction was incorrect, then you must discuss where your reasoning went wrong. If your prediction was correct, then you should review your reasoning and discuss how this lab served to confirm your knowledge of the basic physical concepts.

In this lab, we determined frequency of oscillations of the cart attached to two different springs. The calculated results agree with experimental results, within our range of uncertainty. We concluded that the theoretical prediction based on Newton's Second Law and Hook's Law adequately describes the experimental situation. We also found some small effect of friction. However, for initial times, friction can be neglected.

After you have compared your predictions to your measured results, it is helpful to use an alternative measurement to check your theory with the actual data. This should be a short exercise demonstrating to yourself and to your TA that you understand the basic physics behind the problem. Most of the lab problems are written to include alternative measurements.

Our obtained results also agree with stop watch measurement of oscillation period: T=1.5 s $\Rightarrow f = 0.67$ Hz.

This alternative measurement is within experimental uncertainty of our other experimental and predicted results. Finally, we noticed that theoretical predictions of above experiment coincide with side-by-side configuration.

References

List books, journals or any other resources that you used to write your lab report.

1. Tipler, Paul A. Physics for Scientists and Engineers. 4th, W. H. Freeman: 1999. p 180, 928, 930.

Graph Title

Lab VIII-3 Oscillation frequency with 2 springs

X - Prediction Equation

u(t)=0.000 + 0.000t

Y - Prediction Equation

u(t)=0.000 + 0.000t

X - Fit Equation

u(t) = 0.000 + 18.500 cos (0.000 + 4.680 t + 0.000

Y - Fit Equation

u(t)=2.000 + 0.000t

Vx - Prediction Equation

u(t) = 0.000 + 86.580 sin (0.000 + 4.680 t + 0.000 t^2)

Vy - Prediction Equation

u(t)=0.000 + 0.000t

Vx - Fit Equation

u(t) = 0.000 + -92.000 sin (-0.150 + 4.680 t + 0.000
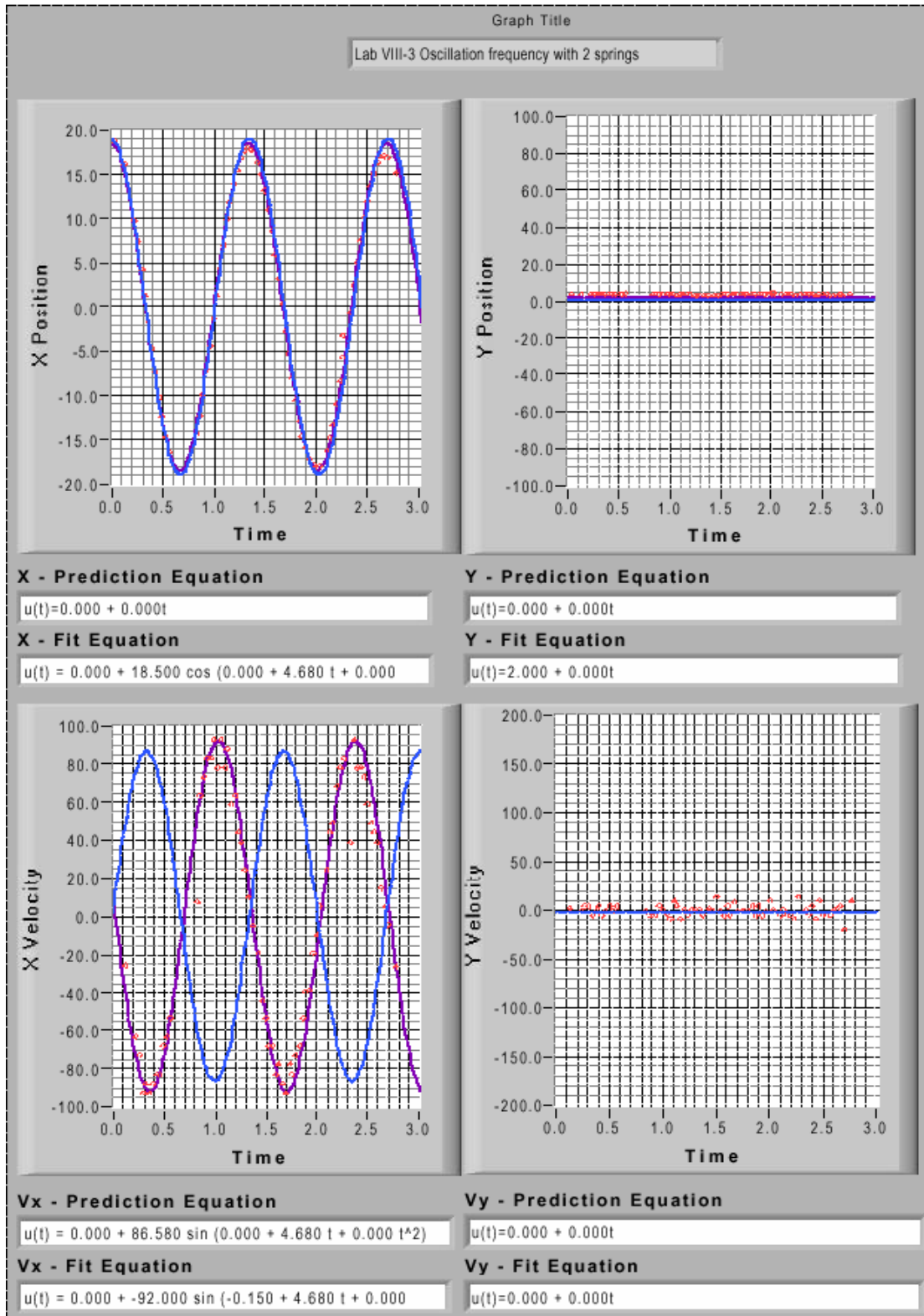
Vy - Fit Equation

u(t)=0.000 + 0.000t

Fig. 5. Experimental results from video analysis.